25 $\sum_{n=0}^{\infty}\left(\frac{\sqrt{x}}{2}-1\right)^n$ ~~ratio test~~ or geo.

$1+\left(\frac{\sqrt{x}}{2}-1\right)+\left(\frac{\sqrt{x}}{2}-1\right)^2+\dots$

$a=1$ $r=\frac{\sqrt{x}}{2}-1$

$\left|\frac{a}{1-r}\right|$

$\left|\frac{\sqrt{x}}{2}-1\right|<1$

sum:

$-1<\frac{\sqrt{x}}{2}-1<1$

$\frac{1}{1-\left(\frac{\sqrt{x}}{2}-1\right)}$

$0<\frac{\sqrt{x}}{2}<2$

$\frac{1}{2\left(2-\frac{\sqrt{x}}{2}\right)}$

$0<\sqrt{x}<4$

$0<x<16$

i.o.c.

$\frac{2}{4-\sqrt{x}}$

sum

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27. $\sum_{n=0}^{\infty}\left(\frac{x^2-1}{3}\right)^n$ geo

$a=1$

$r=\frac{x^2-1}{3}$

i.o.c. : $\left|\frac{x^2-1}{3}\right|<1$

$-1<\frac{x^2-1}{3}<1$

$-3<x^2-1<3$

$\sqrt{x^2}=|x|$

$-2<x^2<4$

$0<|x|<2$

$-2<x<2$

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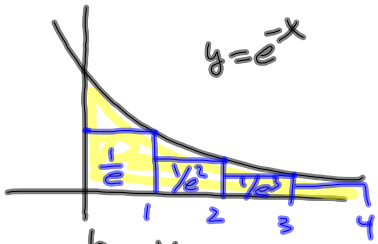
9.5a More Test for Convergence

The Integral Test

$y=e^{-x}$



yellow area:

converge or diverge?

If converge, to what

converges to 1

$$\lim_{b\to\infty}\int_0^b e^{-x}\,dx$$

$$\lim_{b\to\infty} -e^{-x}\Big|_0^b = \lim_{b\to\infty}\left(-e^{-b}--e^{-0}\right) = 1$$

$$\sum_{n=1}^{\infty}\frac{1}{e^n} = \frac{1}{e}+\frac{1}{e^2}+\frac{1}{e^3}+\dots$$

converge or diverge?

geo.

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$$\boxed{\sum a_n \;\&\; \int_1^{\infty} f(x)\,dx \quad \text{behave the same}}$$

f(x) corresponds to a_n

$$f(n)=a_n$$

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Does the series converge?

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}} \qquad \int_1^{\infty} \frac{1}{x\sqrt{x}}\,dx = \int_1^{\infty} \frac{1}{x^{3/2}}\,dx$$

converges

converges

$$\lim_{b\to\infty} \int_1^{b} x^{-3/2}\,dx = 2$$

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Harmonic Series and P-Series

diverges

$$1 + \frac{1}{2} + \frac{1}{3} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n}$$

$$\int_1^{\infty} \frac{1}{x}\,dx = \lim_{b\to\infty} \ln x \Big|_1^b = \lim_{b\to\infty} (\ln b - \ln 1) = \infty$$

diverges

p series

$$\boxed{\sum_{n=1}^{\infty} \frac{1}{n^p} \quad \text{if } p > 1 \text{ series converges} \quad \text{if } p \le 1 \text{ series diverges}}$$

ex.

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ conv. $p = 2 > 1$

$\sum_{n=1}^{\infty} \frac{1}{n^3}$ conv $p = 3 > 1$

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ div $p = \frac{1}{2} < 1$

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Limit Comparison Test LCT

If a_n & b_n grow at the same rate

then $\sum a_n$ & $\sum b_n$ behave the same

If $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = L$ & $0 < L < \infty$

then a_n & b_n grow at same rate

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Do the following series converge or diverge?

$$\sum_{n=1}^{\infty} \frac{2n+1}{(n+1)^2} \qquad \sum \frac{2n}{n^2} = \sum \frac{2}{n} = 2\sum \frac{1}{n}$$

diverges by LCT

same rate? yes

2x harmonic

diverges

$$\lim_{n\to\infty} \frac{\frac{2n+1}{(n+1)^2}}{\frac{2n}{n^2}} = \lim_{n\to\infty} \frac{2n+1}{(n+1)^2} \cdot \frac{n^2}{2n}$$

$$= \lim_{n\to\infty} \frac{2n^3 + n^2}{2n^3 + \dots} = 1$$

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$\sum_{n=1}^{\infty} \frac{1}{2^n - 1} < \sum_{n=1}^{\infty} \frac{1}{2^n}$ geo $r = \frac{1}{2}$

converges converges

same rate yes

$\lim_{n\to\infty} \frac{1}{2^n - 1} \cdot \frac{2^n}{1} = 1$

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$\sum_{n=2}^{\infty} \frac{3n+2}{n^3 - 2n}$ $\sum_{n=2}^{\infty} \frac{3n}{n^3} = \sum_{n=2}^{\infty} \frac{3}{n^2}$ p series $p = 2$

also converges by LCT man

converges

same rate? yes

$\lim_{n\to\infty} \frac{3n+2}{n^3-2n} \cdot \frac{n^2}{3}$

$\lim_{n\to\infty} \frac{3n^3+2n^2}{3n^3-6n} = 1$

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